Midterm 1 October 11, 2017 Duration: 50 minutes

*This test has **4 questions** on **7 pages**, each worth 10 points, for a total of 40 points.*

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations. Answers without justifications will not be marked, except question #3 where the answer alone is sufficient.
- Continue on the closest blank page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **No aids of any kind are allowed**, including: documents, cheat sheets, electronic devices of any kind (including calculators, phones, etc.)

First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Signature: _____

Question:	1	2	3	4	Total
Points:	10	10	10	10	40
Score:					

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   (i) speaking or communicating with other examination candidates, unless otherwise authorized;
   (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
   (iii) purposely viewing the written papers of other examination candidates;
   (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

5 marks 1. (a) Find the area of the triangle with vertices given by the three points $A = (0, 1, 2)$, $B = (1, 0, 3)$, and $C = (1, 1, 4)$.

Answer:

5 marks (b) Find the angle the plane containing the triangle $\triangle ABC$ makes with the (x, y)-plane. Please leave your answer in "calculator-ready format". -

Answer:

5 marks 2. (a) Let $w(x,y) = \frac{x^2}{x+2y}$. Compute the partial derivatives w_x and w_y.

Answer:

Answer:

Can w be a solution of the differential equation $2w_x - w_y = \frac{cx}{x+2y}$?

- ☐ No, not possible.
- ☐ Yes, for any value of c.
- ☐ Yes, but only for $c =$ ☐.

5 marks

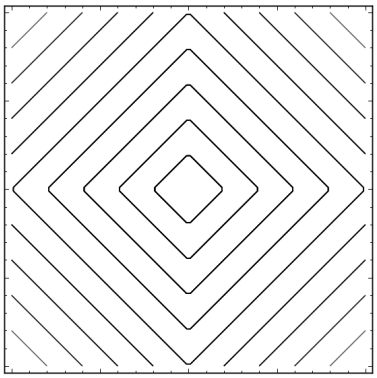
(b) Suppose u is a function of two independent variables x and y. Specifically, let $u(x, y) = x^3y^2$. What is $u_x(1, 2)$?

Answer:

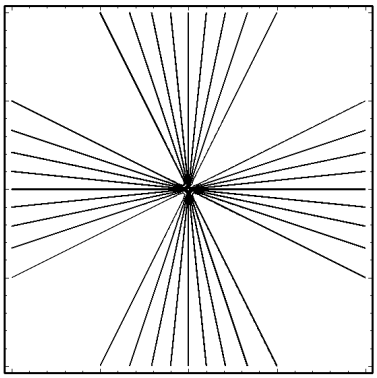
Suppose further that v is also a function of x and y and $\ln v = u - 3x$. (Here "ln" means the natural logarithm, log base e.) What is $v_x(1, 2)$?

Answer:

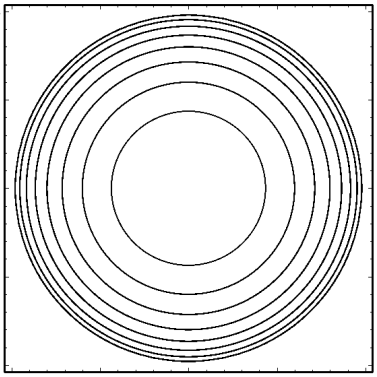
10 marks 3. Consider the following contour plots:



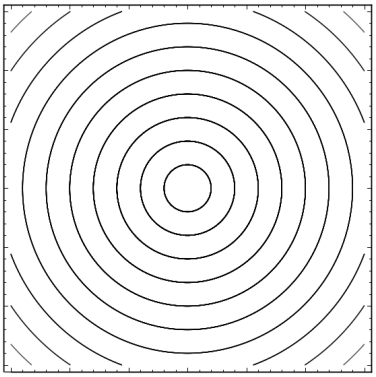
Answer:



Answer:



Answer:



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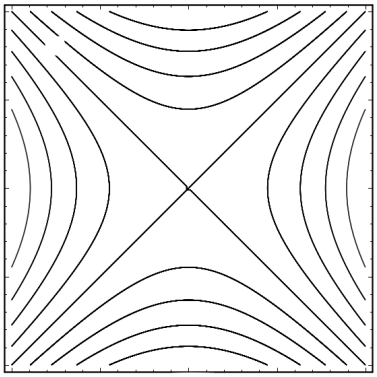
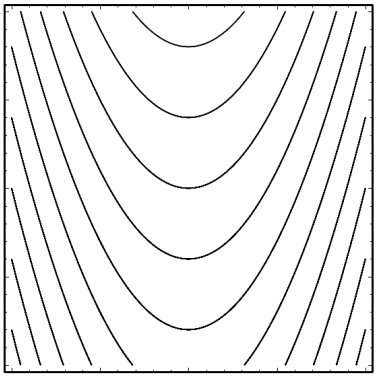


Answer:



Answer:

In the box below each plot, list the name(s) of *all* possible functions which could generate that contour plot:

$$A(x,y) = x^2 - y^2, \qquad D(x,y) = \sqrt{1 - x^2 - y^2}, \qquad G(x,y) = y - 2x^2,$$

$$B(x,y) = \frac{xy}{x^2 + y^2}, \qquad E(x,y) = \sqrt{1 - x^2 - y^2} + \frac{\sin(10x)}{e^{100}}, \qquad H(x,y) = \sqrt{x^2 + y^2},$$

$$C(x,y) = |x| + |y|, \qquad F(x,y) = x^2 + y^2, \qquad I(x,y) = \max(x,y).$$

4. We consider the surface given by the equation $z = x^4 - y^2$. We ask a non-engineer to find the equation of the tangent plane to this surface at the point $(x_0, y_0, z_0) = (2, 3, 7)$. The response is:

$$z = 4x^3\,(x-2) - 2y\,(y-3)$$

2 marks (a) Without any calculation, explain why this cannot be the correct answer.

3 marks (b) Find the correct answer.

Answer:

2 marks (c) Give an expression for the total differential $\mathrm{d}z$ (at any point (x, y)).

Answer:

3 marks (d) At time $t = 0$, an ant leaves the point $(x_0, y_0, z_0) = (2, 3, 7)$ and walks around on the surface, returning to $(2, 3, 7)$ at $t = 2\pi$. The ant's path, when viewed from above, appears to be a unit circle centered at $(x, y) = (1, 3)$ in the xy-plane. Give an expression for the ant's position as a function of time.

Answer:

This page has been left blank for your rough work and calculations.