1. Consider the surface described by the equation $e^{xyz-1} + 3 = x^2 + y^2 + z^2$.

(a) Find a function F(x, y, z) such that (a, b, c) is on the surface if and only if we have F(a, b, c) = 0. Compute the gradient of F.

4 marks

4 marks

(b) Give the equation of the tangent plane to the surface at $(x_0, y_0, z_0) = (1, 1, 1)$ in the form x + by + cz + d = 0. Note that we require the coefficient next to x to be 1.

2. The sugar concentration in an infinite 3D compost bin is given by the equation $S(x, y, z) = (x + y + 2z)^2$. A fruit fly is at position (1, 1, 1).

2 marks

(a) Compute the directional derivative of S at (1, 1, 1) in the direction of $\vec{u} = (-1, 0, 1)$.

2 marks

(b) The fruit fly is feeling a little hungry. In what (unit) direction should the fly move if it wishes to increase the concentration of sugar in the fastest possible way?

Name:

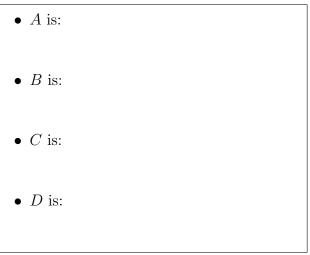
6 marks

2 marks

3. (a) A differentiable function z = f(x, y) is unknown, but an alien supercomputer gave us precise values of f(x, y) and its derivatives on points A, B, C and D.

point	f	f_x	f_y	f_{xx}	f_{yy}	f_{xy}
A	1	0	0	1	0	-5
B	1	0	-2	3	8	4
C	2	0	0	3	3	-2
D	2	0	0	3	3	6

For points A, B, C and D determine whether they are a local minimum, local maximum, a saddle point, or none of the above.



Student-No.:

2 marks

(b) (Bonus marks) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function such that f(1,0) = f(0,0) = 0. Show that there exists $\langle a,b \rangle$ such that $\nabla f(a,b)$ is orthogonal to $\langle 1,0 \rangle$. *Hint:* Define g(t) = f(t,0). Combine the 1D mean value theorem and the chain rule to conclude. This page has been left blank for your rough work and calculations.