1. Consider the function $f(x, y) = x^2y + e^{x-y}$ for parts (a),(b),(c) and (d).

4 marks (a) Compute f_x, f_y, f_{xy} and f_{yx}.

4 marks (b) Compute the equation of the tangent plane to the graph of $z = f(x, y)$ at $(2, 2, 9)$.

2 marks (c) **Use the previous part** to approximate $f(2.1, 1.9)$.

4 marks

(d) Find a point (a, b, c) in the graph of $z = f(x, y)$ such that its tangent plane has the equation $3x - z = 1$. *Hint: there is a solution such that* $a = 1$.

6 marks

2. (a) You maneuver a spaceship in a three dimensional space. At time t the position of the spaceship is given by the vector $\langle x(t), y(t), z(t)\rangle$. A proton star at the origin emits radiation in such a way that the perceived radiation at a point in space is given by the equation $R(x, y, z) = e^{-(x^2+y^2+z^2)}$.

Assume that at time $t = 0$ the position of the spaceship is $\langle x(0), y(0), z(0)\rangle = \langle 1, 1, 1\rangle$ and that its velocity is $\langle x'(0), y'(0), z'(0)\rangle = \langle 1, 2, -4\rangle$. Determine the rate of change of the perceived radiation by the spaceship at time $t = 0$.

2 marks

(b) **(Bonus marks)** Construct an example of a function $f(x, y)$ such that every level curve is a single line of the form $c = 2x + y$ for some $c \in \mathbb{R}$ but whose graph is NOT a plane.

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